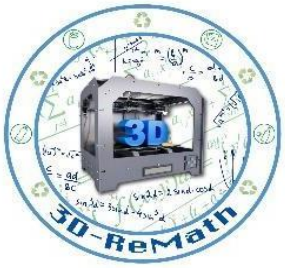


# 3D printing technology aims students understanding maths and recycling procedure

**Project code: 2019-1-EL01-KA201-062914**

*Erasmus+ Call: 2019 - KA2 - Cooperation for Innovation and the Exchange of Good Practices - Strategic Partnerships for school education*

UNIVERSITY OF THE AEGEAN

2nd Primary School of Chios

ISA13 Sarzana

AEM Agrupamento de Escolas de Marrazes

QmLab Waste Management Laboratory PrivaSI

## Intellectual Output 2: Curricula No2

## Mathematics: Mathematical Thinking/Sets & Combinations

ΙΔΡΥΜΑ ΚΡΑΤΙΚΩΝ ΥΠΟΤΡΟΦΙΩΝ ΙΚΥ

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# 1. Introduction

This a class for students from 6 to 14 years old, which aims to help students develop mathematical thinking by using the concepts of sets and combinations, and 3d printing.

**Topic**

This scenario is an innovative way to raise **mathematical thinking** of young students using the **technology of 3D Printing**. The goal of this curricula is to aim students at problem formulation and problem solving, by using simple exampled. The topics under examination in this curricula are **Sets** and **Combinations**

The approach of the subject is done according to the methodology of the guided explanatory learning. Exploration is an approach to learning that is related to general types of thinking mechanisms and high-level cognitive skills.

**Tools Used:**

- Two **questionnaires** of ten items were developed in order to define the topics of mathematics in which students face difficulties. The first questionnaire was addressed at students aged between 6 Each question focused directly on a specific mathematical topic. All questions had multiple answers but only one was correct.
  The sample size was 930 students from the three countries: Greece, Italy and Portugal. The sample size was estimated through normal distribution and it was representative for each school.
  Answers were collected and analyzed using SPSS Package 21.00.
- **Story telling technic** will be used to describe a story with many questions, that students have to answer and through this cognitive quiz they will understand the concepts of sets and combinations.
- **3D Printing** technology was also used, as an innovative methodology to aim students at understanding sets and combinations. Students will also learn to use 3D Printers. Students will use **Tinker Cad** or **Sketch Up Software** in order to design 3D schemes, like number or letters and then they use 3D printers to print “these objects” in order to understand now they could group letters or numbers based on a specified criterion
- **Worksheets** will be given to students during the course
- **Power Point Presentations**

# 2. Goals, Learning Outcomes, Requirements

**Main idea:**



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The main purpose of the curriculum is **the set and combinations of** objects and the methods for grouping them into clusters. The goal of the project is to help students to define a criterion based on which the objects are placed to groups and then is to present in a clear and precise way l differences between sets and combinations.

**Value added:**
In this process of exploratory learning the use of 3D printing technology can be of great value. The students can experiment with objects like letters or numbers and use criteria like color or size in order to group letters and numbers. By using 3D printing technology and print letters and numbers of different size and different color then they separate the solid objects to groups and learn the corresponding mathematical concepts.

**Required helping material and tools:**

- A **3D Printer** per four students.
- A **school textbook** that provides theory and exercises
- **Worksheets** both for the course delivery and for homework.

## Learning Outcomes

- Distinguish things among different criteria
- Realize mathematical concepts, such as sets and combinations
- Perform simple calculations
- Exploit mathematical thinking
- Learn the structure of Pascal Triangle

**Expected results:**

In terms of **mathematical achievement,** Students will be able:

- to **understand** the concept of Sets and Combinations
- to **distinguish** objects based on a criterion
- to **identify differences** between sets and combinations of objects
- **to learn** the different uses of Pascal Triangle

In terms of **teaching process** the students will:

- **Follow** the teacher's **instructions**.
- **Ask questions** and **give answers** to them.
- **Complete** the worksheet.

In terms of **3D Printing**, students will be able

- to **use** a 3D Printer



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- to **search** and find 3D models at .stl files
- to **import** a file at a 3D printer
- to **slice** a model
- to **print** a model to play with solid object

**The courses are organized as a mathematical game. However, teachers should have a strong theoretical background in the mathematical terms involved. For this purpose, they are encouraged to study the following supplementary on-line material related to sets, combinations, and Pascal Triangles**



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## 3. Course Outline

**Duration**
The duration of the proposed course is 8-10 hours, depends on students' educational level, age of students, number of students at class and time for printing circles and parts of circles

**Material**
Teachers could use the following educational material

- PowerPoint Presentations
- Videos
- Worksheets
- 3D Printers

# Course 1: Introduction to Sets

✓ **Mathematical Concepts**
- Definition of Sets
- Sets of things
- Notation of Sets
- Criteria for sets
- Sets of Numbers

**Definitions**

**Set:** in a simplify way set is a collection of objects, which may be or not be of a mathematical nature. They may be functions or numbers, or letters
Cantor wrote that a set, is a collection of definite, distinguishable objects of perception or thought conceived as a whole. The objects are called elements or members of the set

**Equal Sets**: If two sets have the same elements, then they are equal

**Null Set**: Is the set that it has no elements

**Known Sets from Mathematics** (only for students under 11 years old:
- Set of **Natural** Numbers $\mathbb{N}$={0,1,2,3,…..}
- Set of I**nteger** Numbers Z={….-2,-1,0,1, 2,…}
  This set consists of the counting numbers with their corresponding negative values and zero
- Set of **Rational Numbers** Q: set of ratios of integers. Any number that belong to this group can be written aas the ratio p/q of any two integers, where q<>0



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- Set of **Irrational numbers**: This set contains all numbers that can not be written as ratio of two integers
- Set of **Real Numbers** :=all numbers

**Subset**: A set A is called a subset of set B if all elements of A are also elements of set B, i.e, A={1,2,3,4,5,6} and B={1,2,3,4,5,6,7,8,9}

**Criteria:** Use everyday examples to explain the notion “well-defined”. The importance of criteria in mathematical thinking

✓ **Power Point Presentation**
“Introduction to Sets”
“Sets and Activities”

✓ **Videos**

https://www.youtube.com/watch?v=6T16rd15R2E
*https://www.youtube.com/watch?v=sMb4Aupn-ek*
*https://www.youtube.com/watch?v=_9Wvu-R04go*



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## Course 2: Sets and Activities

✓ **Mathematical Concepts**

- Understanding with activities all definitions about set at Course 1

Some more definitions

- https://en.wikipedia.org/wiki/Set_(mathematics)
- https://www.math-only-math.com/sets.html

✓ **Power Point Presentation**

"Sets and Activities"

✓ **Activities**

1. The teacher should provide with 15 balls of different colors and sizes, as follows: 1 white small ball, 3 white big balls, 2 green small balls, 4 green big balls, 2 black small balls and 3 black big balls. The students are asked to perform the following tasks:
    a. Create the set of all green balls
    b. Create the set of all big balls
    c. Create the set of all balls that are not green
    d. Create the set of all balls that are green and big
    e. Create the set of all balls that are white and not big
    f. Create the set of all balls that are not white, not green and not black

***Teachers with students from 6-10 years old should skip the theoretical part, start with the activity and just present the game of applying a criterion***

2. Use the sets defined in the previous activity and determine which sets are subsets to another

✓ **Activity at 3D Printer**

**Print numbers and letters**

- Print numbers 1,2,3,4,5 in three different colors
- Print letters A,B, C, D, E in three different colors
- Group the above solid objects in sets according to color
- Group the above 15 printed numbers based on number
- Group the above 15 letters based on "letter"



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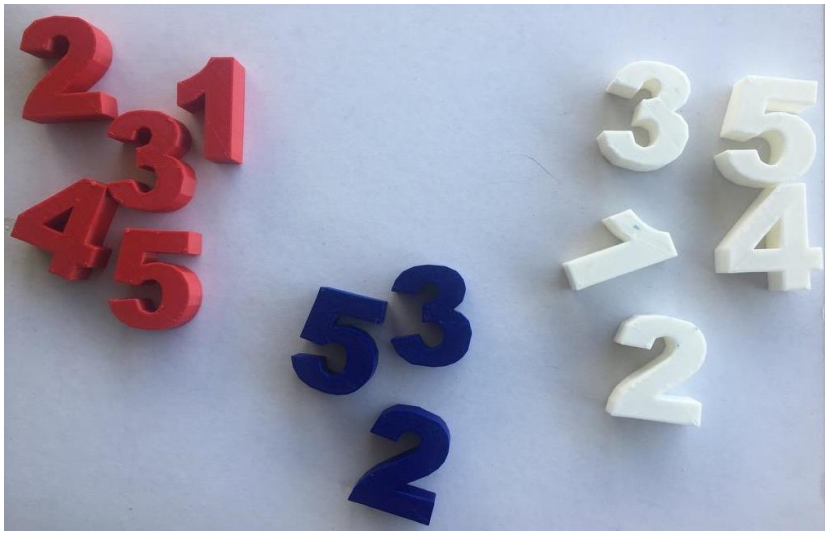
**Answer**

- Print letters 1,2,3,4,5 in three different colors

Group the above solid numbers in sets according to color





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- Group the above printed numbers based on number

- Print letters A,B, C, D in different colors



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- Group the above printed letters based on “color”



✓ **Videos**
https://www.youtube.com/watch?v=fes92vSBTg4



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## Course 3-4: Introduction to Mathematical Thinking

✓ **Mathematical Concepts**
- Introduction to Mathematical Thinking
- Mathematical representation of sets and elements

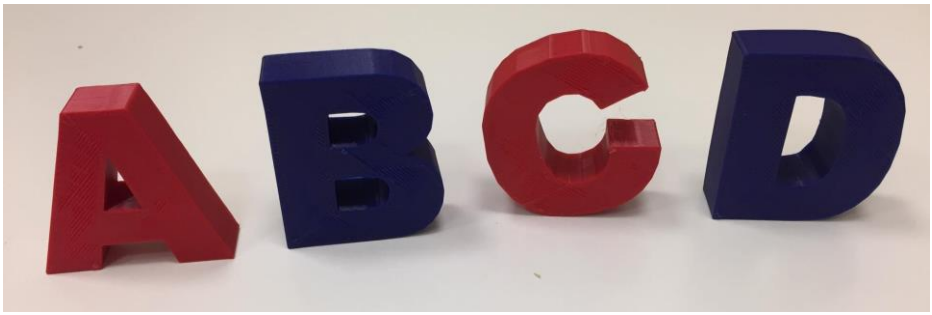
Mathematical thinking is a lot more than just being **able to do arithmetic or solve algebra problems**. It is a whole way of looking at things, stripping them down to their essentials, whether it's numerical, structural or logical and then analyzing the underlying patterns. **Math is about patterns**. When we are teaching a mathematical method, we are showing something that happens all the time, something that happens in general. Getting students to see these underlying structures, whether it's in a math problem, in society, or in nature, is one of the reasons that studying mathematics is so worthwhile. It transforms math from drudgery to artistry
https://drvcourt.wordpress.com/2016/07/08/what-is-mathematical-thinking/

✓ **Power Point Presentation**
"Mathematical Thinking"

✓ **Activity at 3D Printer**

**Print letters**
- Print letters A,B, C, D
- Use these letters in order to prepare the activity described at pptx
- Define in how many ways we can choose 2 out of 4 letters

All Letters




Choose 2 out of 4

We choose letter A with all possible combinations



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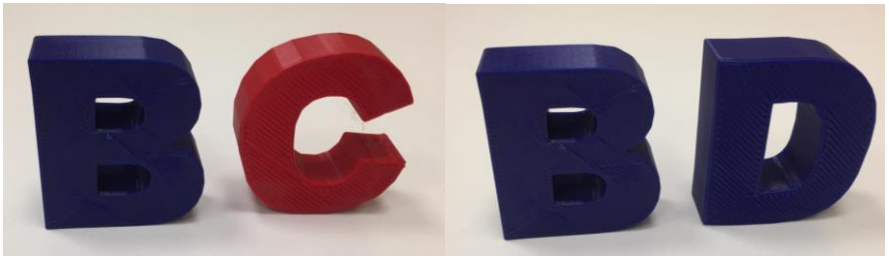
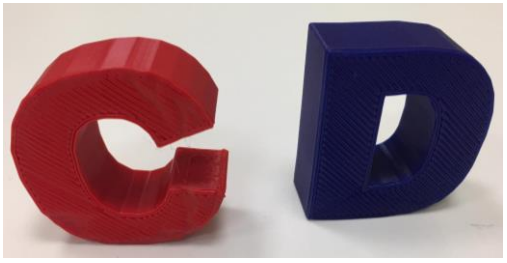
We choose letter B with all possible combinations



Note that the set {B,A}, has already be chosen above (Set {A,B}) and we don't choose it again

We choose letter C with all possible combinations

Note that sets {C,A} and {C,B} , have already be chosen above (Sets {A,C} and {B,C}) and we don't choose them again



✓ **Activity**

*Distinction between the use of small and capital letters*



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All 15 balls of activity 1 should now be labelled with the small letters a-o. The students are also provided with 6 labeled bags with the capital letters A-F. For each one of the six tasks of Activity 1, the students are asked to fill each bag with the appropriate content and describe the resulting set using an appropriate notation on the white board.

Teachers with students from 6-10 years old should skip mathematical notation, start with the activity and label the bags and balls by whole words, i.e. ball1, ball2, …, ball15, BAG1, BAG2, …, BAG 6



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# Course 5: From Sets to Subsets

✓ **Mathematical Concepts**

- Introduction to Subsets
- Elements of Sets and Subsets

**Definition**

- If all elements of set 1 are present in set 2 then we say that set 1 is a subset of set 2. We know that a set is a well-defined collection of numbers, alphabets, objects, or any items. If set 1 = {A,B,C} and set 2 = {A,B,C,D,E,F} we can say that set 1 is a subset of set 2 since all the elements in set 1 are present in set 2.
- **Subset**: A set A is called a subset of set B if all elements of A are also elements of set B, i.e, A={1,2,3,4,5,6} and B={1,2,3,4,5,6,7,8,9}

✓ **Power Point Presentation**

"Sets and Subsets"

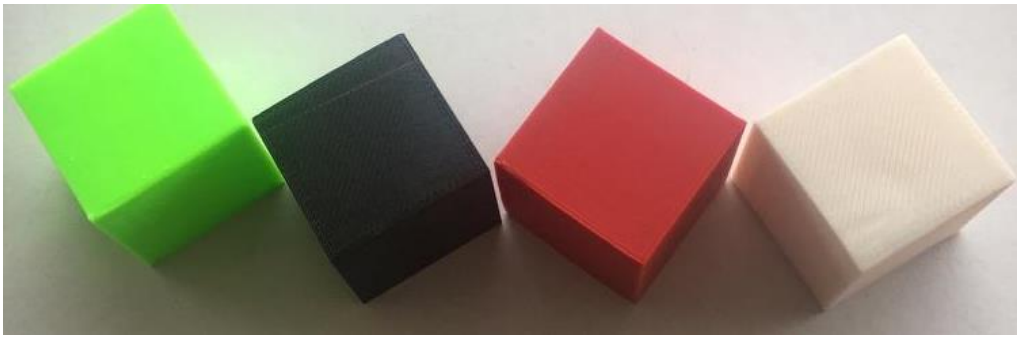
✓ **Videos**

https://study.com/academy/lesson/subsets-in-math-definition-examples-quiz.html
https://www.youtube.com/watch?v=sMb4Aupn-ek

✓ **Activity at 3D Printer**

**Print spheres**

- Print cubes in 4 different colors
- How many different sets can be made by choosing two colored cubes from the four colored cub?



✓ **Activity**

A. In a room there are two blonde women, one black-haired woman, two black-haired men and one blonde man. How many elements does the set of the black-haired



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people in the room has? Perform a mathematical representation of the above set and name it set D.

B. We consider all people contained in set D. In how many ways could one choose two different people among them?

***Teachers with students from 6-10 years old should avoid using the words set and elements***



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# Course 6-7: Combinations

✓ **Mathematical Concepts**

- Introduction to Combinations
- Examples with Combinations
- Differences between Sets and Combinations
- Combinations with repetition
- Combinations without repetitio

**Definition**

- A **combination** is a mathematical technique that determines the number of possible arrangements in a collection of items where the order of the selection does not matter. In combinations, you can select the items in any order.
- Combinations can be confused with **permutations**. However, in permutations, the order of the selected items is essential. For example, the arrangements *ab* and *ba* are equal in combinations (considered as one arrangement), while in permutations, the arrangements are different.

Some more definitions

- https://en.wikipedia.org/wiki/Combination
  https://corporatefinanceinstitute.com/resources/knowledge/other/combination/
- https://www.mathsisfun.com/definitions/combination.html

✓ **Power Point Presentation**

"Combinations"

✓ **Videos**

- https://www.onlinemathlearning.com/combinations-probability-2.html
- https://www.turtlediary.com/video/combinations.html

✓ **Activity at 3D Printer**

Print 4 cubes in different colors



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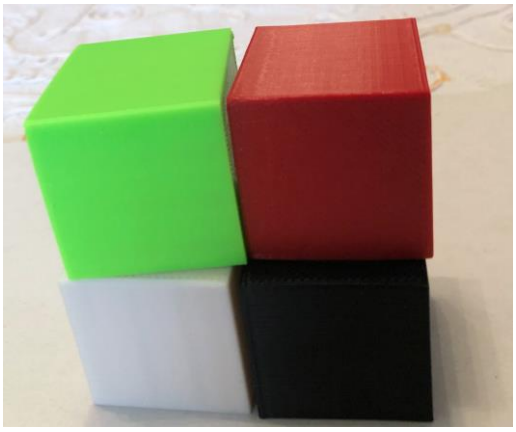
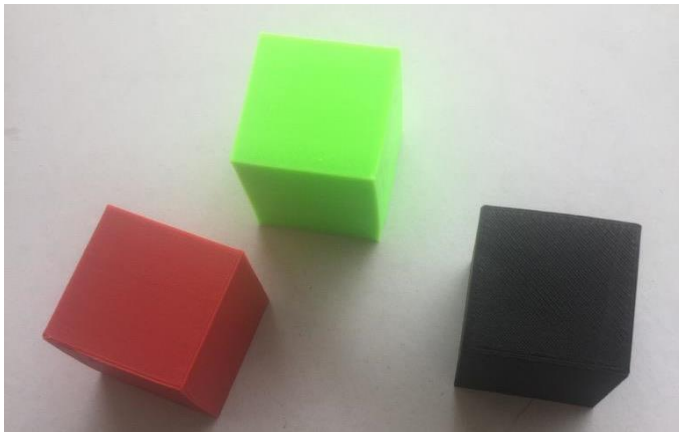


✓ **Activities**

The students are asked to print 5 cubes. One white, one green, one black , one red ball and one yellow. Then they asked the following questions.

1. Use the white, the green and the black cube. In how many ways you may choose
    a. One cube
    b. Two cubes
    c. Three cubes

**Answer**



Choose 1 cube





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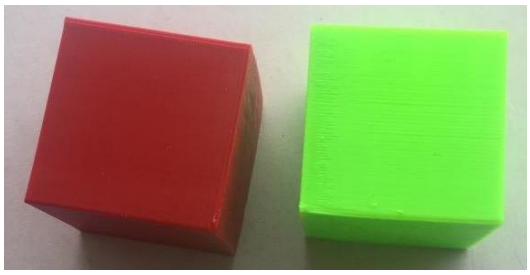
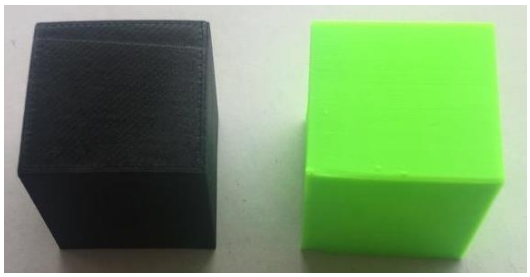
3 different ways

Choose 2 cubes





3 different ways

Choose 3 cubes



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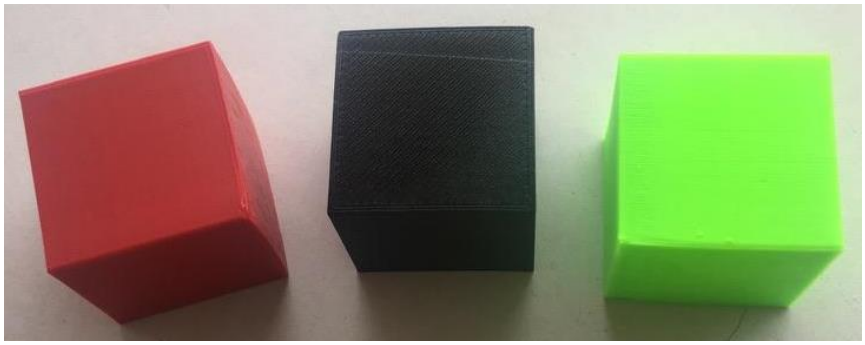


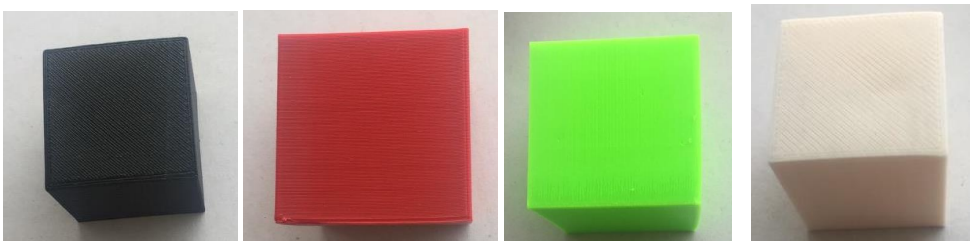
1 way

Total 7 ways

2. Use the white, the green, the balck and the red cubel. In how many ways you may choose (repetition is allowed)
    a. One cube
    b. Two cubes
    c. Three cubes
    d. Four cubes

**Answer**

Choose 1 cube



Four different ways

Choose 2 cubes



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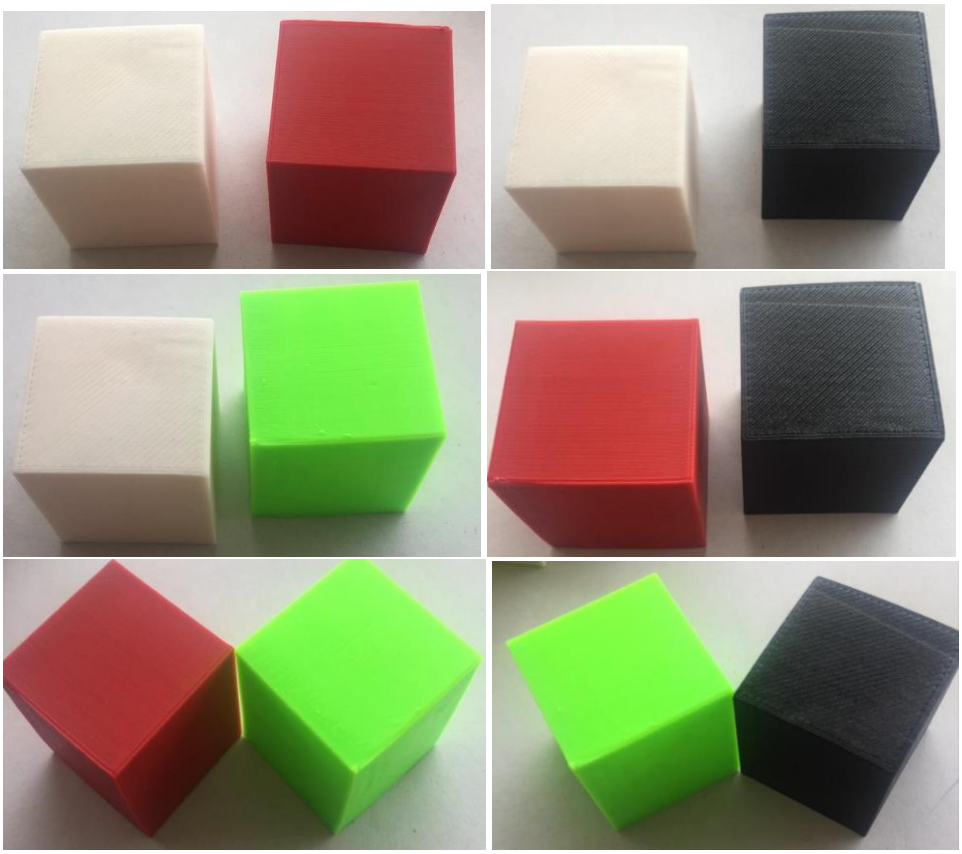
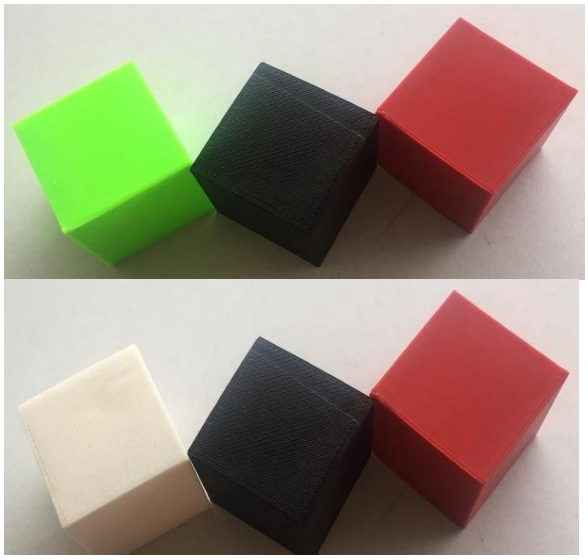


Six different ways

Choose 3 cubes





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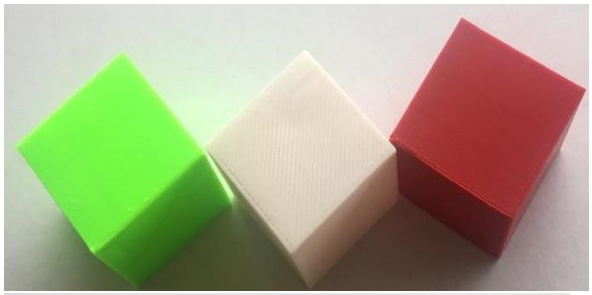
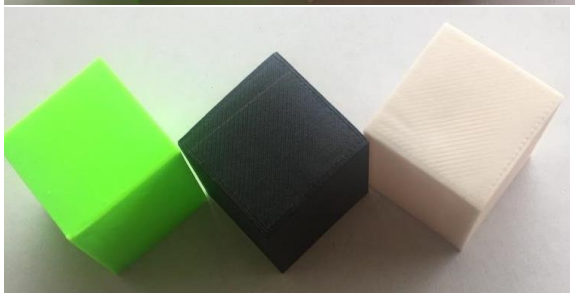
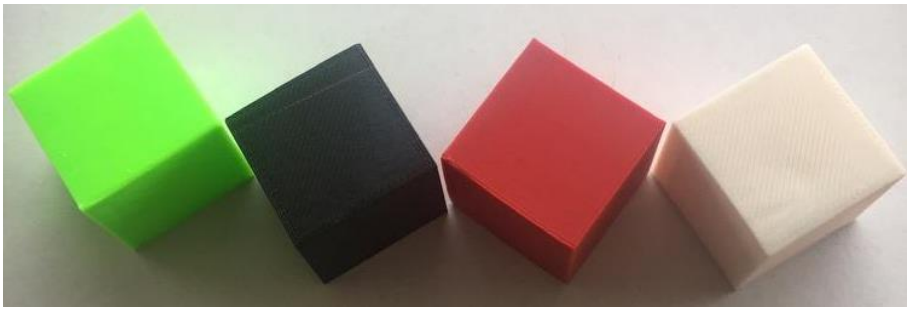




Four different ways

Choose 4 cubes



1 way

Total 15 ways

3. Use all five balls. In how many ways you may choose
    a. One cubes
    b. Two cubes
    c. Three cubes
    d. Four cubes
    e. Five cubes

## Course 8: Pascal Triangle



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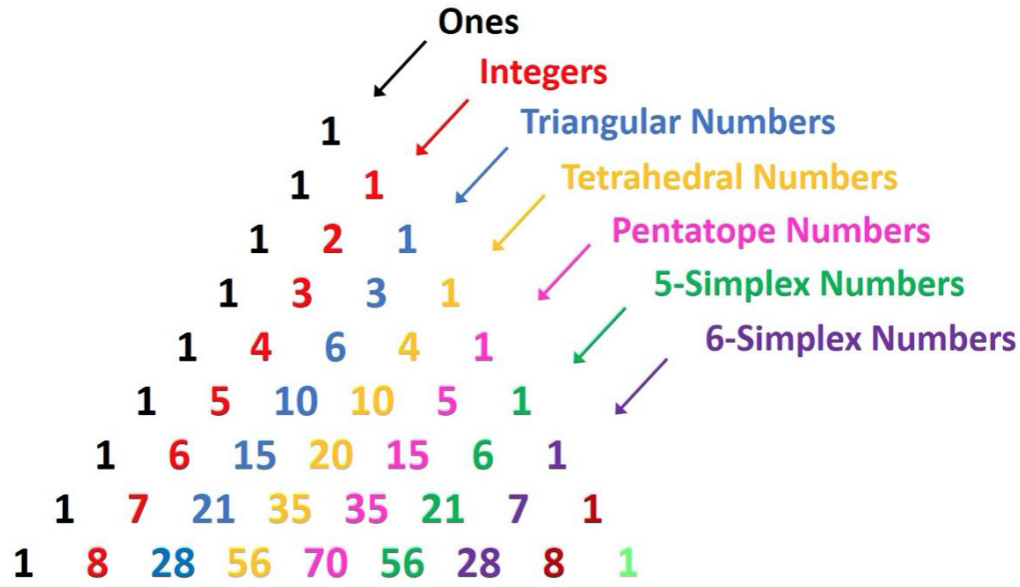
**(only for students 11-14)**

✓ **Mathematical Concepts**

- What is a Pascal Triangle
- The structure of Pascal Triangle

**Definition**




- Some more definition on Pascal Triangles
  https://en.wikipedia.org/wiki/Pascal%27s_triangle
  https://mathworld.wolfram.com/PascalsTriangle.html

- Pascal triangle contains all binomial coefficients in order. So if you have five objects and wants to calculate in how many unique ways you can choose 2 objects, you can use Pascal triangle. Start on the top left of the triangle, move down 5 spaces, along 1'sand then across 2 spaces. You get the answer of 10



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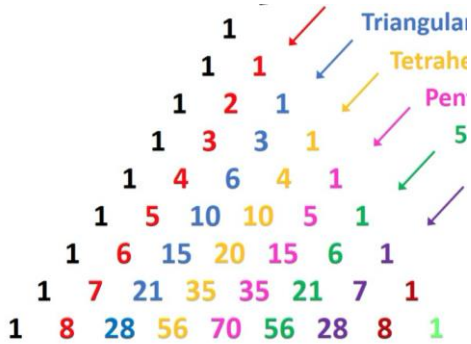


✓ **Video**

- https://www.youtube.com/watch?v=XMriWTvPXHI
- https://www.youtube.com/watch?v=YUqHdxxdbyM

✓ **Activity at 3D Printer**

Print numbers in different colors in order to construct a Pascal Triangle

✓ **Activity**

Suppose that we must make timetable for 300 students, trying to avoid class clashing and all the rest. If we have 8 modules to choose from and each student picks 4, then how many unique combinations are there?



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