



# 3D printing technology aims students understanding maths and recycling procedure

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UNIVERSITY OF THE AEGEAN



Intellectual Output 2: Mathematics' Curricula

Deliverable Code: Fractions



Erasmus+

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## 1. Introduction

This a class for 5<sup>th</sup> and 6<sup>th</sup> grade of primary school, 1<sup>st</sup> and 2<sup>nd</sup> grade of secondary school, who need to review **fractions** using 3d printing and filament of recycled materials.

### Topic

This scenario is an alternative way to teach fraction to students, using the **technology of 3D Printing**. The approach of the subject is done according to the methodology of the guided explanatory learning.

Exploration is an approach to learning that is related to general types of thinking mechanisms and high-level cognitive skills.

### Tools Used:

- Two **questionnaires** of ten items were developed in order to define the topics of mathematics in which students face difficulties. The first questionnaire was addressed at students aged between 6 Each question focused directly on a specific mathematical topic. All questions had multiple answers but only one was correct.  
The sample size was 930 students from the three countries: Greece, Italy and Portugal. The sample size was estimated through normal distribution and it was representative for each school.  
Answers were collected and analyzed using SPSS Package 21.00.
- **3D Printing** technology was also used, as an innovative methodology to aim students at understanding fractions. Students will learn to use 3D Printers. Students will use **Tinker Cad** or **Sketch Up Software** in order to design 3D schemes, like circles and quarters and then they use 3D printers to print “pizzas” and parts of pizzas, in order to understand the concept of fractions.
- **Worksheets** will be given to students during the course
- **Power Point Presentations**

## 2. Goals, Learning Outcomes, Requirements

### Main idea:

The main purpose of the curriculum is **the examination/ exploration of** fractions and consider them as a representation of the result of dividing two natural numbers. The second aim is to understand that equivalent fractions are different representations of the same result.

### Value added:

In this process of exploratory learning the use of 3D printing technology can be of great value. The students can experiment with different representations of the same concept by changing the parameters (divide circle to two, three, six, eight parts) and by transforming the mathematical concepts to solid objects using 3D printing technology. Thus, the students print

different parts of a circle, different color and they add, subtract, and multiply these parts assumptions about the given topic.

**Required helping material and tools:**

- A **3D Printer** per four students.
- A **school textbook** that provides theory and exercises
- **Worksheets** both for the course delivery and for homework.

**Learning Outcomes**

- Compare and order fractions and mixed numbers
- Represent equivalent fractions
- Identify the division statement that represents a fraction
- Apply techniques of converting fractions into other, equivalent ones.
- Simplify the numerator and denominator of a fraction by dividing them with the same divisor
- Add and subtract fractions having like and unlike denominators

**Expected results:**

In terms of **mathematical achievement**, Students will be able:

- to **understand** the concept of percentages and the fractions
- to **distinguish** proper and improper fractions
- to **find** equivalent fractions.
- to **compare** fractions
- to **convert** fractions into other, equivalent ones.
- to **use** cross-products to **check** if two fractions are equivalent.
- to **add, subtract** and **multiply** fractions.

In terms of **teaching process** the students will:

- **Follow** the teacher's **instructions**.
- **Ask questions** and **give answers** to them.
- **Complete** the worksheet.

In terms of **3D Printing**, students will be able

- to **use** a 3D Printer
- to **search** and find 3D models at .stl files
- to **import** a file at a 3D printer
- to **slice** a model
- to **print** a model to play with solid object



### 3. Course Outline

#### Duration

The duration of the proposed course is 8-10 hours, depends on students' educational level, age of students, number of students at class and time for printing circles and parts of circles

#### Material

Teachers could use the following educational material

- PowerPoint Presentations
- Videos
- Worksheets
- 3D Printers

#### Syllabus of courses

##### Course 1: Introduction to Fractions 1

✓ **Mathematical Concepts**

Basic Concepts such as

- Entire object (ONE) or Whole Object
- Percentage 100%
- Numerator
- Denominator

✓ **Videos**

Fraction <https://www.youtube.com/watch?v=zQuUNE50JnM>

✓ **Power Point Presentation**

“Introduction to Fractions”

## Course 2: Introduction to Fractions 2

✓ **Mathematical Concepts**

- Mixed Numbers (Whole number and fraction)
- Proper and improper fractions

✓ **Videos**

Fraction <https://youtu.be/jqWqSjgMAtw>

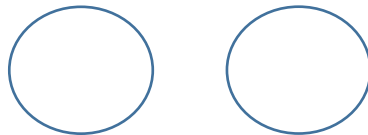
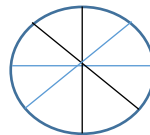
✓ **Power Point Presentation**

“Introduction to Fractions”

✓ **Activity**

Divide a circle into 8 equal parts

- Color 6 of 8 parts
- Explain WHOLE and fraction
- What do I have to add to  $\frac{6}{8}$  to make it equal to 1
- Write down the equivalent fraction
- Divide each circle into fourths. Color 1 ( $\frac{3}{4}$ )



## Course 3: Equivalent Fractions

### ✓ Mathematical Concepts

- Like and Unlike Fractions
- Equivalent Fractions
- Fractions Simplifications
- Irreducible Fractions

### ✓ Video

Equivalent Fraction: <https://www.youtube.com/watch?v=GVSUcvq-40U>

### ✓ Power Point Presentation

“Equivalent Fractions”

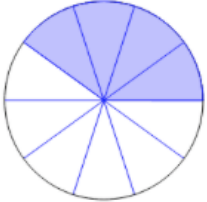
### ✓ Activity at 3D Printer

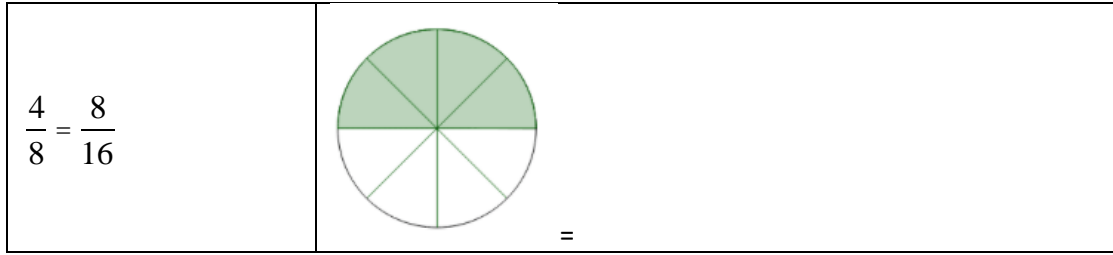
#### Print four cubes (Like fractions)

- Play with the cubes
- Place cubes in such a way in order to create a large cube
- Large cube is considered as a whole
- Take 2 cubes out of four, write the corresponding fraction
- Take 3 cubes out of four and write the corresponding fraction

### ✓ Activity

Based on the equal fractions of 1<sup>st</sup> column, draw the corresponding circles

Equal fractions	Equal pies
$\frac{2}{4} = \frac{1}{2}$	
$\frac{2}{3} = \frac{6}{9}$	
$\frac{4}{10} = \frac{2}{5}$	
$\frac{3}{6} = \frac{6}{12}$	





## Course 4: Fraction of parts/Compare fractions

### ✓ **Mathematical Concepts**

- Unlike Fractions (same numerator/different denominator or different numerator and different denominator)
- Compare fractions with same denominator
- Compare fractions with different denominator
- Order fractions

### ✓ **Video**

Compare and order fractions <https://www.youtube.com/watch?v=nH7s9SIjwus>

### ✓ **Power Point Presentation**

“Compare Fractions”

### ✓ **Activity at 3D Printer**

**Print 1 circle pan or a pizza**

**Print parts from a pizza**

- Print 4 parts of a pizza
- Print 6 parts of a pizza
- Print 8 parts of a pizza

**Play with the parts of the pizza**

- Compare the parts
- Take 3 out of 8 parts of the pizza and write down the corresponding fraction
- Take 4 out of six parts of the pizza and write down the corresponding fraction
- Take 2 out of 4 parts of the pizza and write down the corresponding fraction
- Take 3 out of 6 parts of the pizza and write down the corresponding fraction
- Compare all the above fractions



✓ **Activity**

1. Jorge had two pizzas, one pizza “Margarita” and one pizza “Peperoni”. Lucia ate  $\frac{1}{2}$  of Margarita pizza and Patricia ate  $\frac{1}{3}$  of Peperoni pizza. Lucia claimed that she ate less than Patricia. Do you agree?
  
2. Veronica had 24 chocolates. She gave  $\frac{1}{2}$  to Pollina and  $\frac{1}{6}$  to Rosa
  - Write down the whole in terms of 8 and in terms of 6
  - How many chocolates did she give to Pollina?
  - How many chocolates did she give to Rosa?
  - How many chocolates did she have left?

3. Compare the following fractions by using printed pies

I.  $\frac{1}{2}, \frac{1}{4}$

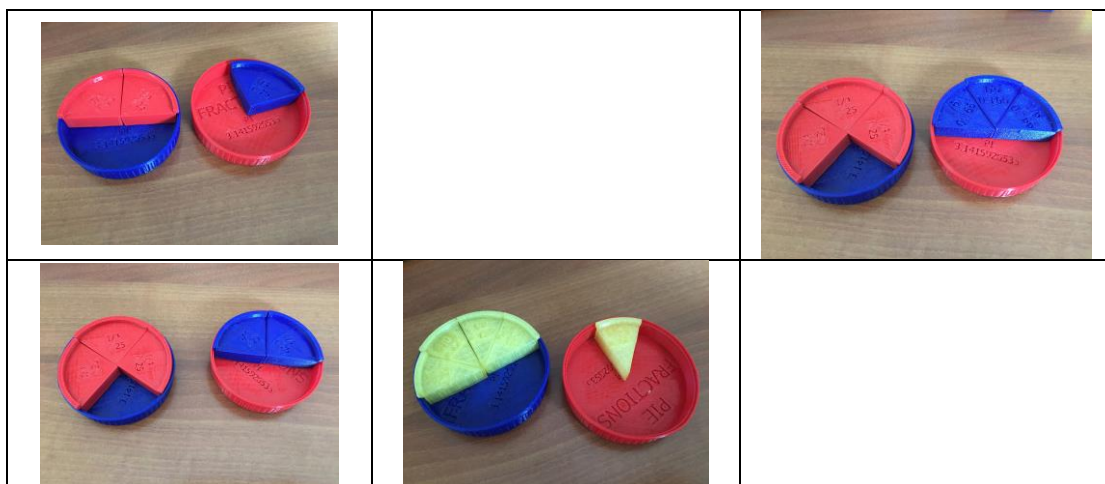
II.  $\frac{5}{6}, \frac{5}{7}$

III.  $\frac{3}{4}, \frac{3}{6}$

IV.  $\frac{3}{4}, \frac{2}{4}$

V.  $\frac{1}{8}, \frac{4}{8}$

VI.  $\frac{3}{7}, \frac{1}{7}$



±

## Course 5: Fractions, Wholes Patterns

✓ **Mathematical Concepts**

- Concept of whole. Many and different shapes could be defined as a whole
- Measurement
- Deductive Reasoning

✓ **Video**

Patterns of Fractions <https://www.youtube.com/watch?v=dkBSmdBI-Os>

✓ **Power Point Presentation**

“Patterns of Fractions”

✓ **Activity at 3D Printer**

**Print 1 hexagon prism**

**Print triangular prisms as parts of hexagon prism**

- Print 6 triangles
- Take 3 or 1 or 2 out of 6 triangular prism of the prism and write down the corresponding fraction

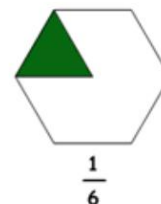
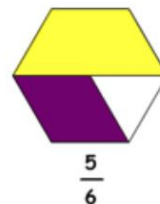
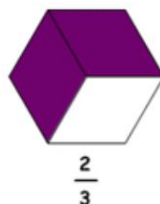
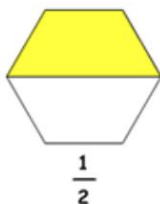
**Print 1 cuboid rectangular prism**

**Print 1 triangular prism, trapezoid prism**

- Use the above solid objects, and find the pattern to develop a hexagon prism

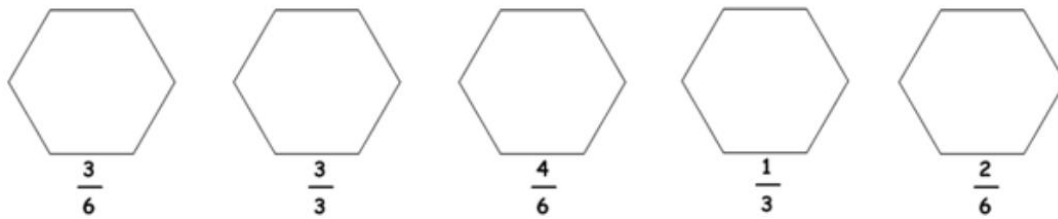
✓ **Activity**

1. If hexagon is whole (=1) and pattern blocks show the fractions

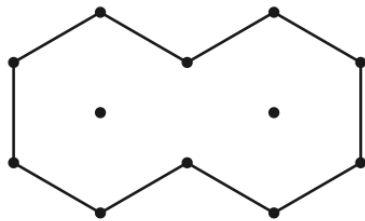


Color pattern blocks to show the corresponding fractions

Source of pictures: Internet

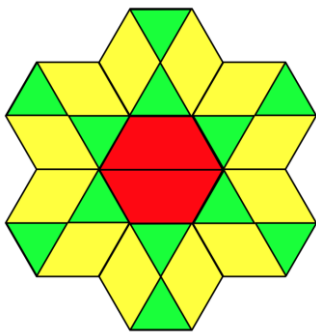


2. Two hexagons together are one whole. Draw line segments to divide each into trapezoids, rhombuses and triangles. Write a number model to show how the parts add up the whole



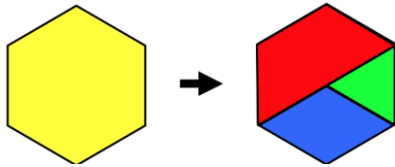
Source of pictures: Internet

3. What fraction of the design is red? What fraction is green? Justify your answer



Source of pictures: Internet

4. Spyros used three different pattern blocks to cover the yellow block below. If the yellow block is 1, then what addition sentence can you use to express John’s design?



Source of pictures: Internet

5. If hexagon is whole and equals "3", what is the value of the following

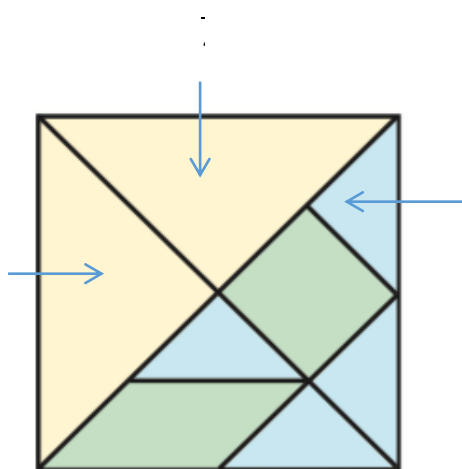
$$\text{Hexagon} = 3$$

Source of pictures: Internet


## Course 6: Fractions of squares

### ✓ **Mathematical Concepts**

Square is whole (Square=1)



2 blue triangles and 1 green square  
Green Square= 2 blue triangles

Thus,  $\frac{1}{4}$  Square= 4 blue triangles

1 blue triangle=  $\frac{1}{4} \left( \frac{1}{4} \text{ Square} \right) = \frac{1}{16}$  Square

### ✓ **Video**

Fraction of square:

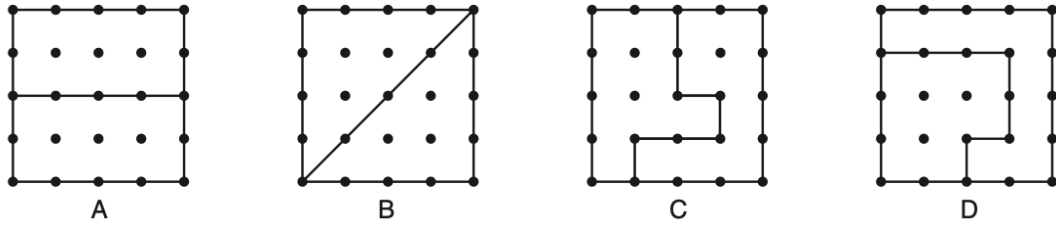
- [https://www.youtube.com/watch?v=nD9soP3j41A&list=RDCMUCHnj59g7jezwTy5GeL8EA\\_g&start\\_radio=1&t=31](https://www.youtube.com/watch?v=nD9soP3j41A&list=RDCMUCHnj59g7jezwTy5GeL8EA_g&start_radio=1&t=31)
- [https://www.youtube.com/watch?v=6K\\_j4Cj7mVo](https://www.youtube.com/watch?v=6K_j4Cj7mVo)

### ✓ **Power Point Presentation**

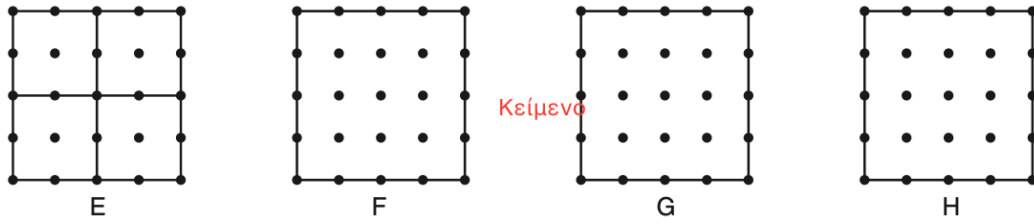
“Fraction of squares”

### ✓ **Activities**

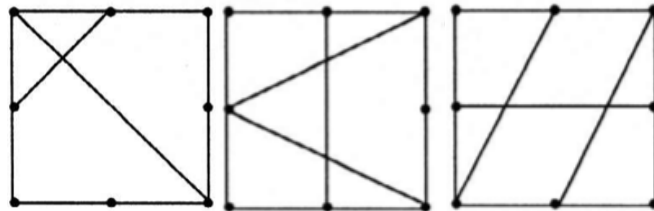
1. The following squares are divided in half in different way, by using straightedges and dots



Square E is divided to fourths. Divide square F, G, and H into fourths, each in a different way



2. Dots at the following squares represent the middle of the sides of squares. Write the fraction of the square in the following picture





## Course 7: Add and Subtract Fractions

### ✓ **Mathematical Concepts**

- Add and Subtract Fractions with like dominators
- Review of Least Common Multiple
- Add and Subtract Fractions with unlike dominators
- Add and Subtract Mixed Fractions and simplify the results

### ✓ **Video**

- <https://www.youtube.com/watch?v=QgizFohvJo0>
- <https://www.youtube.com/watch?v=tDQipFjAoT8>
- Addition of fractions: <https://www.youtube.com/watch?v=1thX9Qml0Ks>

### ✓ **Power Point Presentation**

“Add Subtract Fractions”

### ✓ **Activity at 3D Printer**

**Print 1 circle pan or a pizza**

**Print parts from a pizza**

- Print 4 parts of a pizza
- Print 6 parts of a pizza
- Print 8 parts of a pizza

Let's think: Pizza is a circle of  $360^\circ$

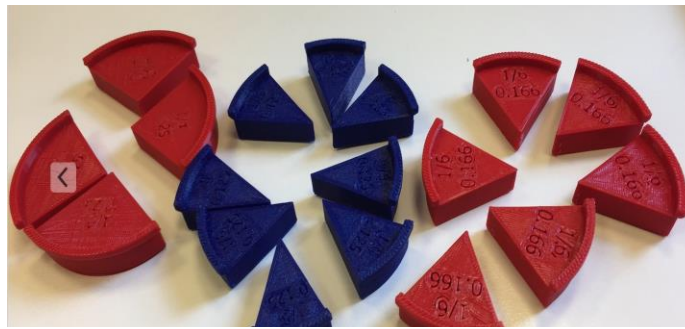
- If we separate the circle into 4 parts each part is of  $90^\circ$
- If we separate the circle into 6 parts each part is of  $60^\circ$
- If we separate the circle into 8 parts each part is of  $45^\circ$

These parts are fractions with different denominator

**Play with the parts of the pizza**

- Place to the circle pan 2 parts of  $45^\circ$  and 1 part of  $90^\circ$ . Write down in a fraction form the result
- Place to the circle pan 1 part of  $90^\circ$  and 2 parts of  $60^\circ$ . Write down in a fraction form the result
- Place to the circle pan 1 part of  $90^\circ$ , 1 part of  $60^\circ$  and 1 part of  $45^\circ$ . Write down in a fraction form the result

- Play with the different parts of pizza



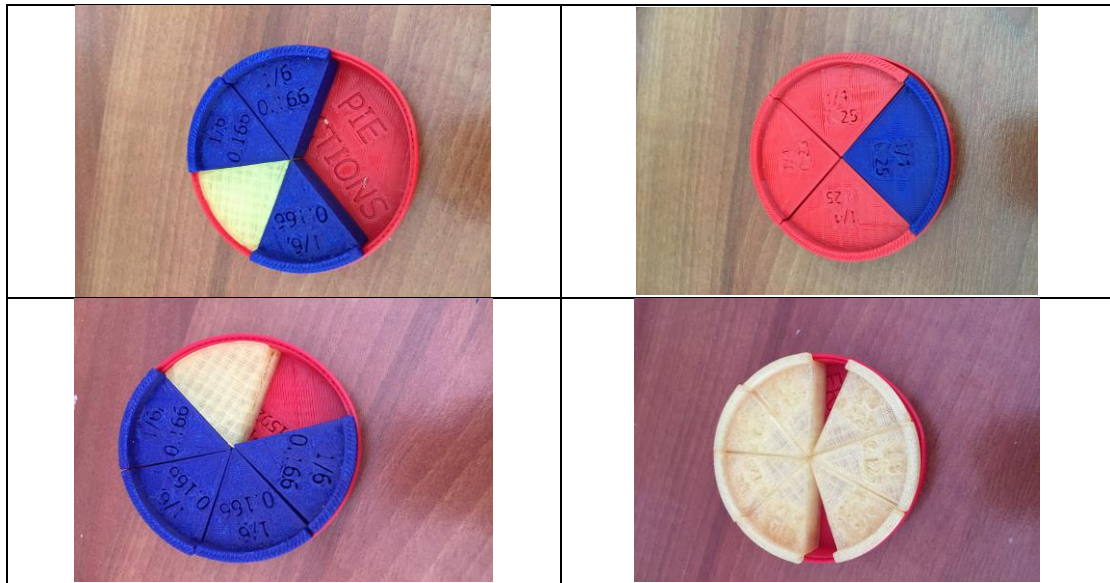
✓ **Activities**

- Use printed pies of 4,6 and 8 pieces and calculate the following  
**Addition**

$$\frac{1}{6} + \frac{3}{6} = \quad \frac{1}{4} + \frac{3}{4} = \quad \frac{1}{2} + \frac{1}{2} =$$

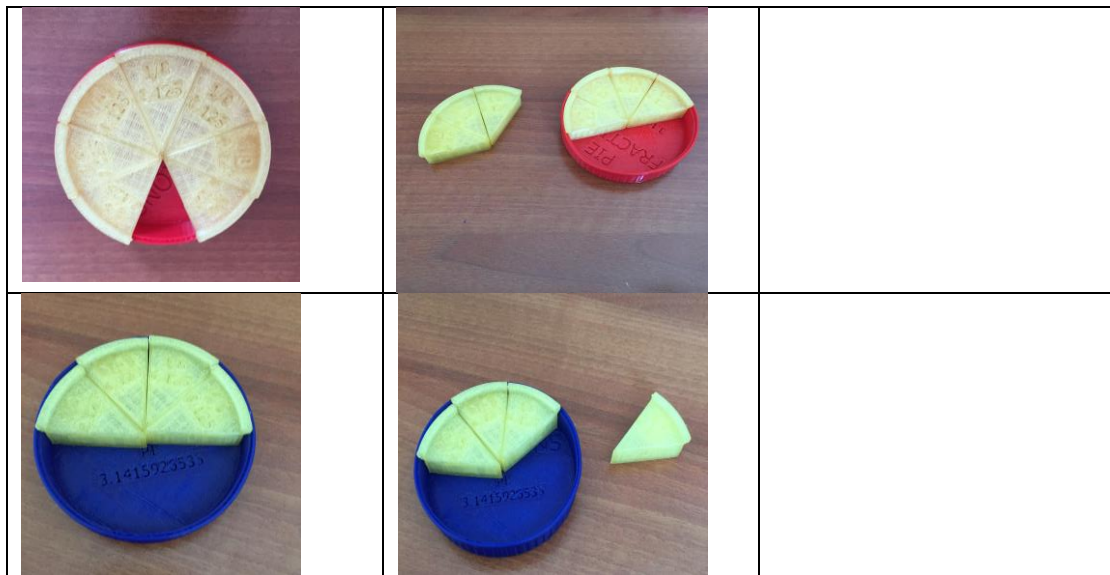
$$\frac{4}{6} + \frac{1}{6} = \quad \frac{2}{4} + \frac{1}{4} = \quad \frac{4}{8} + \frac{3}{8} =$$

$$\frac{1}{8} + \frac{5}{8} = \quad \frac{3}{8} + \frac{4}{8} = \quad \frac{5}{8} + \frac{3}{8} =$$



2. Use printed pies of 4,6 and 8 pieces and calculate the following

Subraction



$$\frac{3}{4} - \frac{1}{4} =$$

$$\frac{2}{4} - \frac{1}{4} =$$

$$1 - \frac{1}{4} =$$

$$\frac{3}{6} - \frac{1}{6} =$$

$$\frac{5}{6} - \frac{2}{6} =$$

$$\frac{4}{6} - \frac{2}{6} =$$

$$\frac{4}{8} - \frac{1}{8} =$$

$$\frac{7}{8} - \frac{3}{8} =$$

$$\frac{5}{8} - \frac{3}{8} =$$

$$\frac{3}{8} - \frac{1}{4} =$$

$$\frac{2}{6} - \frac{2}{4} =$$

$$\frac{7}{8} - \frac{3}{4} =$$

$$\frac{3}{6} - \frac{1}{4} =$$

$$\frac{5}{6} - \frac{2}{4} =$$

$$\frac{4}{6} - \frac{2}{8} =$$

$$\frac{4}{8} - \frac{1}{4} =$$

$$\frac{7}{8} - \frac{3}{6} =$$

$$\frac{5}{6} - \frac{1}{8} =$$

## Course 8: Multiply and divide fractions

### ✓ **Mathematical Concepts**

- Rules for multiplying fractions
- Multiply fractions by whole numbers
- Multiply mixed fractions
- Rules for dividing fractions

### ✓ **Video**

- [https://www.youtube.com/watch?v=LU3R2JE5c\\_U](https://www.youtube.com/watch?v=LU3R2JE5c_U)
- <https://www.youtube.com/watch?v=x6xtezhuCZ4>

### ✓ **Power Point Presentation**

“Multiply Fractions”

### ✓ **Activities**

$$\frac{1}{2}x\frac{3}{8} = \quad \frac{1}{4}x\frac{5}{6} = \quad \frac{7}{3}x\frac{2}{6} =$$

$$\frac{1}{3}x\frac{2}{6} = \quad \frac{3}{4}x\frac{3}{6} = \quad \frac{6}{5}x\frac{5}{6} =$$

$$\frac{1}{2}x\frac{2}{9} = \quad \frac{2}{7}x\frac{3}{8} = \quad \frac{6}{8}x\frac{4}{3} =$$

$$\frac{4}{7}x\frac{2}{8} = \quad \frac{8}{7}x\frac{3}{8} = \quad \frac{11}{9}x\frac{3}{2} =$$

$$\frac{1}{9}x\frac{3}{6} = \quad \frac{11}{7}x\frac{4}{3} = \quad \frac{7}{4}x\frac{3}{5} =$$